

Neutrino Mass and Flavour Models¹

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Abstract. We survey some of the recent promising developments in the search for the theory behind neutrino mass and tri-bimaximal mixing, and indeed all fermion masses and mixing. We focus in particular on models with discrete family symmetry and unification, and show how such models can also solve the SUSY flavour and CP problems. We also discuss the theoretical implications of the measurement of a non-zero reactor angle, as hinted at by recent experimental measurements.

Keywords: Neutrino mixing, family symmetry

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INTRODUCTION

It has been one of the long standing goals of theories of particle physics beyond the Standard Model (SM) to predict quark and lepton masses and mixings. With the discovery of neutrino mass and mixing, this quest has received a massive impetus. Indeed, perhaps the greatest advance in particle physics over the past decade has been the discovery of neutrino mass and mixing involving two large mixing angles commonly known as the atmospheric angle $\theta_{23} = 43.1^\circ \pm 4^\circ$ and the solar angle $\theta_{12} = 34.5^\circ \pm 1.4^\circ$ where the current one sigma ranges to typical global fits are displayed [1]. There is a 2σ hint for a non-zero reactor mixing angle $\sin^2 \theta_{13} = 0.02 \pm 0.01$ [2] which gives the one sigma range $\theta_{13} = 8^\circ \pm 2^\circ$. The largeness of the two large lepton mixing angles contrasts sharply with the smallness of the quark mixing angles, and this observation, together with the smallness of neutrino masses, provides new and tantalizing clues in the search for the origin of quark and lepton flavour which has led to a resurgence of interest in this subject [3].

It is a striking fact that current data on lepton mixing is (approximately) consistent with the so-called tri-bimaximal (TB) mixing pattern [4],

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P_{Maj}, \quad (1)$$

where P_{Maj} is the diagonal phase matrix involving the two observable Majorana phases. However there is no convincing reason to expect exact TB mixing, and in general we expect deviations. These deviations can be parametrized by three parameters r, s, a

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defined as [5]:

$$\sin \theta_{13} = \frac{r}{\sqrt{2}}, \quad \sin \theta_{12} = \frac{1}{\sqrt{3}}(1+s), \quad \sin \theta_{23} = \frac{1}{\sqrt{2}}(1+a). \quad (2)$$

Global fits of the conventional mixing angles [1, 2] can be translated into the 1σ ranges

$$0.14 < r < 0.24, \quad -0.05 < s < 0.02, \quad -0.04 < a < 0.10. \quad (3)$$

Note in particular that the central value of r is now 0.2 which corresponds to a 2σ indication for a non-zero reactor angle as discussed in [2].

Clearly a non-zero value of r , if confirmed, would rule out TB mixing. However it is possible to preserve the good predictions that $s = a = 0$, by postulating a modified form of mixing matrix called tri-bimaximal-reactor (TBR) mixing [6],

$$U_{TBR} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}}(1-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}} \end{pmatrix} P_{Maj}. \quad (4)$$

NEUTRINO FLAVOUR SYMMETRY

Let us expand the neutrino mass matrix in the diagonal charged lepton basis, assuming exact TB mixing, as $M_{TB}^V = U_{TB} \text{diag}(m_1, m_2, m_3) U_{TB}^T$ leading to (absorbing the Majorana phases in m_i):

$$M_{TB}^V = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T \quad (5)$$

where $\Phi_1^T = \frac{1}{\sqrt{6}}(2, -1, 1)$, $\Phi_2^T = \frac{1}{\sqrt{3}}(1, 1, -1)$, $\Phi_3^T = \frac{1}{\sqrt{2}}(0, 1, 1)$, are the respective columns of U_{TB} and m_i are the physical neutrino masses. In the neutrino flavour basis (i.e. diagonal charged lepton mass basis), it has been shown that the above TB neutrino mass matrix is invariant under S, U transformations:

$$M_{TB}^V = S M_{TB}^V S^T = U M_{TB}^V U^T. \quad (6)$$

A very straightforward argument [7] (see also [8, 9]) shows that this neutrino flavour symmetry group has only four elements corresponding to Klein's four-group $Z_2^S \times Z_2^U$. By contrast the diagonal charged lepton mass matrix (in this basis) satisfies a diagonal phase symmetry T . The matrices S, T, U form the generators of the group S_4 in the triplet representation, while the A_4 subgroup is generated by S, T .

FAMILY SYMMETRY: DIRECT VS INDIRECT MODELS

As discussed in [10], the flavour symmetry of the neutrino mass matrix may originate from two quite distinct classes of models. The first class of models, which we call direct models, are based on a family symmetry $G_f = S_4$, or a closely related family symmetry as discussed below, some of whose generators are directly preserved in the lepton sector

and are manifested as part of the observed flavour symmetry. The second class of models, which we call indirect models, are based on some more general family symmetry G_f which is completely broken in the neutrino sector, while the observed neutrino flavour symmetry $Z_2^S \times Z_2^U$ in the neutrino flavour basis emerges as an accidental symmetry which is an indirect effect of the family symmetry G_f . In such indirect models the flavons responsible for the neutrino masses break G_f completely so that none of the generators of G_f survive in the observed flavour symmetry $Z_2^S \times Z_2^U$.

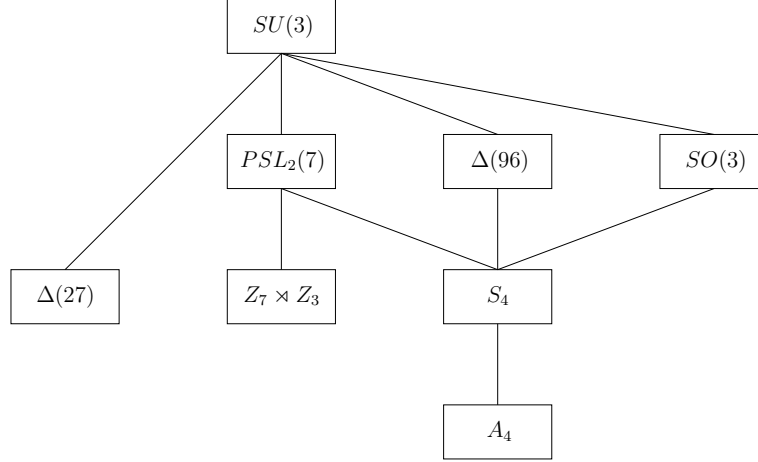


FIGURE 1. Some possible family symmetry groups.

In the direct models, the symmetry of the neutrino mass matrix in the neutrino flavour basis (henceforth called the neutrino mass matrix for brevity) is a remnant of the $G_f = S_4$ symmetry of the Lagrangian, where the generators S, U are preserved in the neutrino sector, while the diagonal generator T is preserved in the charged lepton sector. For direct models, a larger family symmetry G_f which contains S_4 as a subgroup is also possible e.g. $G_f = PSL(2, 7)$ [7]. Some possible family symmetry groups and their relation to S_4 are shown in Figure 1. If the family symmetry of the underlying Lagrangian is smaller, say, $G_f = A_4$ [11], then in some cases this can lead to a direct model where the T generator of the underlying Lagrangian symmetry is preserved in the charged lepton sector, while the S generator is preserved in the neutrino sector, with the U transformation of S_4 emerging as an accidental symmetry due to the absence of flavons in the $\mathbf{1}', \mathbf{1}''$ representations of A_4 [12]. Typically direct models satisfy form dominance [13], and require flavon F-term vacuum alignment, permitting an $SU(5)$ type unification [12]. Such minimal A_4 models lead to neutrino mass sum rules between the three masses m_i , resulting in/from a simplified mass matrix in Eq.5. A_4 may result from 6D orbifold models [14] and recently a 6D $A_4 \times SU(5)$ SUSY GUT model has been constructed [15].

In the indirect models [10] the idea is that the three columns of U_{TB} Φ_i are promoted to new Higgs fields called “flavons” whose VEVs break the family symmetry, with the particular vacuum alignments along the directions Φ_i . In the indirect models the underlying family symmetry of the Lagrangian G_f is completely broken, and the flavour symmetry of the neutrino mass matrix $Z_2^S \times Z_2^U$ emerges entirely as an accidental symmetry, due to the presence of flavons with particular vacuum alignments proportional to the columns of U_{TB} , where such flavons only appear quadratically in effective Majorana

Lagrangian [10]. Such vacuum alignments can be elegantly achieved using D-term vacuum alignment, which allows the large classes of discrete family symmetry G_f , namely the $\Delta(3n^2)$ and $\Delta(6n^2)$ groups [10].

SEE-SAW MECHANISM AND FORM DOMINANCE

It is possible to derive the TB form of the neutrino mass matrix in Eq.5 from the see-saw mechanism in a very elegant way as follows. In the diagonal right-handed neutrino mass basis we may write $M_{RR}^V = \text{diag}(M_A, M_B, M_C)$ and the Dirac mass matrix as $M_{LR}^V = (A, B, C)$ where A, B, C are three column vectors. Then the type I see-saw formula $M^V = M_{LR}^V (M_{RR}^V)^{-1} (M_{LR}^V)^T$ gives

$$M^V = \frac{AA^T}{M_A} + \frac{BB^T}{M_B} + \frac{CC^T}{M_C}. \quad (7)$$

By comparing Eq.7 to the TB form in Eq.5 it is clear that TB mixing will be achieved if $A \propto \Phi_3$, $B \propto \Phi_2$, $C \propto \Phi_1$, with each of $m_{3,2,1}$ originating from a particular right-handed neutrino of mass $M_{A,B,C}$, respectively. This mechanism allows a completely general neutrino mass spectrum and, since the resulting M^V is form diagonalizable, it is referred to as form dominance (FD) [13]. For example, it has recently been shown that the direct A_4 see-saw models [12] satisfy FD [13], where each column corresponds to a linear combination of flavon VEVs.

A more natural possibility, called Natural FD, arises when each column arises from a separate flavon VEV, and this possibility corresponds to the case of indirect models. For example, if $m_1 \ll m_2 < m_3$ then the precise form of C becomes irrelevant, and in this case FD reduces to constrained sequential dominance (CSD)[16]. The CSD mechanism has been applied in this case to the class of indirect models with Natural FD based on the family symmetries $SO(3)$ [16, 17] and $SU(3)$ [18], and their discrete subgroups [19].

PARTIALLY CONSTRAINED SEQUENTIAL DOMINANCE

It is possible to achieve TBR mixing, corresponding to $s = a = 0$ but $r \neq 0$, by a slight modification to the CSD conditions,

$$B = \frac{b}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad A = \frac{c}{\sqrt{2}} \begin{pmatrix} \varepsilon \\ 1 \\ 1 \end{pmatrix}. \quad (8)$$

We refer to this as Partially Constrained Sequential Dominance (PCSD)[6], since one of the conditions of CSD is maintained, while the other one is violated by the parameter ε . Note that the introduction of the parameter ε also implies a violation of FD since the columns of the Dirac mass matrix A, B can no longer be identified with the columns of the MNS matrix, due to the non-orthogonality of A and B . To leading order in $|m_2|/|m_3|$ the mass matrix resulting from Eq.8 leads to TBR mixing where we identify [6],

$$m_1 = 0, \quad m_2 = b^2/M_B, \quad m_3 = a^2/M_A, \quad \varepsilon = re^{-i\delta}. \quad (9)$$

Thus, the TBR form of mixing matrix in Eq.4 will result, to leading order in $|m_2|/|m_3|$. For example, it is straightforward to implement the above example of PCSD into realistic GUT models with non-Abelian family symmetry spontaneously broken by flavons which are based on the CSD mechanism [16, 18, 17]. More generally it is natural to expect vacuum alignments as in Eq.8 from the D-term vacuum alignment associated with the indirect models [10]. The point is that D-term alignment along the direction of the second column of U_{TB} Φ_2 is enforced by the family symmetry G_f , but the other alignments are provided by orthogonality arguments and are therefore intrinsically more model dependent.

FAMILY SYMMETRY \otimes GUT MODELS

In typical Family Symmetry \otimes GUT models the origin of the quark mixing angles derives predominantly from the down quark sector, which in turn is closely related to the charged lepton sector. There are many possibilities for the choice of family symmetry and GUT symmetry. Examples of indirect models along these lines include the Pati-Salam gauge group $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ in combination with $SU(3)$ [18], $SO(3)$ [16, 17], A_4 [20] or Δ_{27} [21]. Examples of direct models along these lines are based on $SU(5)$ GUTs in combination with A_4 [22] or T' [23].

A promising new example of a Family Symmetry \otimes GUT model is the $PSL(2, 7) \times SO(10)$ proposal in [7]. Such a model unifies the three families into a complex $\psi \sim (3, 16)$ representation, with the Higgs $H \sim (1, 10)$, while obtaining the third family Yukawa coupling from a sextet flavon $\chi \sim (6, 1)$ coupling $\chi\psi\psi H$. The other Yukawa couplings arise from two triplet flavons ϕ . The diagrams responsible for the Yukawa couplings are shown in Fig.2.

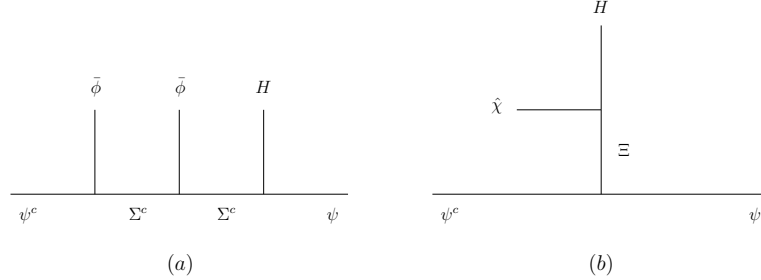


FIGURE 2. Diagrams responsible for the Yukawa couplings in the $PSL(2, 7) \times SO(10)$ model. Diagram (a) shows how two triplet flavon ϕ insertions are responsible for the first and second family Yukawa couplings, while diagram (b) shows how a single sextet flavon χ insertion is responsible for the third family Yukawa couplings.

In order to reconcile the down quark and charged lepton masses, simple ansatze, such as the Georgi-Jarlskog hypothesis [24], lead to very simple approximate expectations for the charged lepton mixing angles such as $\theta_{12}^e \approx \lambda/3$, $\theta_{23}^e \approx \lambda^2$, $\theta_{13}^e \approx \lambda^3$, where $\lambda \approx 0.22$ is the Wolfenstein parameter from the quark mixing matrix. If the family symmetry enforces accurate TB mixing in the neutrino sector, then $\theta_{12}^e \approx \lambda/3$ charged lepton corrections will cause deviations from TB mixing in the physical lepton mixing angles, and lead to a sum rule relation [16, 25, 26], which can be conveniently expressed

as [5] $s \approx r \cos \delta$ where $r \approx \lambda/3$ and δ is the observable CP violating oscillation phase, with RG corrections of less than one degree [27]. Such sum rules can be tested in future high precision neutrino oscillation experiments [28].

Note that in such a GUT-flavour framework, one expects the charged lepton corrections to the neutrino mixing angles to be less than of order $\theta_{12}^e/\sqrt{2}$ (where typically θ_{12}^e is a third of the Cabibbo angle) plus perhaps a further 1° from renormalization group (RG) corrections. Thus such theoretical corrections cannot account for an observed reactor angle as large as 8° , corresponding to $r = 0.2$, starting from the hypothesis of exact TB neutrino mixing.

FAMILY SYMMETRY AND SUSY FLAVOUR/CP PROBLEMS

In SUSY models we not only want to understand the origin of the Yukawa couplings, but also the soft SUSY breaking masses. There are stringent limits from of flavour changing and CP violating processes on the form of these soft masses. These limits may be expressed as bounds on the real and imaginary parts of the mass insertion parameters δ , as recently compiled in [29]. It has been observed that $G_f = SU(3)$ family symmetry implies an approximately universal family structure of the soft masses close to the GUT scale [30] and, with the hypothesis that CP is spontaneously broken by flavon VEVs, this can also lead to suppressed CP violation [30]. These issues have been recently examined in detail in the $G_f = SU(3)$ models which predict tri-bimaximal mixing [29], although the results are also applicable to discrete family symmetry models such as the $G_f = \Delta(27)$ model [21]. The results show that there is a small tension in the model (at least for “reasonable” SUSY masses and parameters) due to the processes $\mu \rightarrow e\gamma$ and the EDMs [29]. However this tension can be completely removed in classes of $G_f = SU(3)$ models based on supergravity [31].

CONCLUSION

We have surveyed some of the recent promising developments in the search for the theory behind neutrino mass and tri-bimaximal mixing, and indeed all fermion masses and mixing. Tri-bimaximal mixing implies a discrete neutrino flavour symmetry $Z_2^S \times Z_2^U$ which can be realized either directly or indirectly via a discrete family symmetry G_f . The direct models are typically based on $G_f = A_4$ or $G_f = S_4$ where the family symmetry generators S and U are preserved in the neutrino sector. The indirect models are typically based on $G_f = \Delta(3n^2)$ or $G_f = \Delta(6n^2)$ where none of the family symmetry generators are preserved in the neutrino sector since they are all broken by the flavons which align along the columns of U_{TB} . However (after the see-saw mechanism) the neutrino sector involves only quadratic combinations of these flavons leading to an accidental discrete neutrino flavour symmetry $Z_2^S \times Z_2^U$.

The type I see-saw mechanism can be elegantly implemented using form dominance in which the columns of the neutrino Yukawa matrix (in the diagonal charged lepton and right-handed neutrino mass basis) are proportional to the columns of U_{TB} . In the direct models these columns are generated from linear combinations of different flavon

alignments, which implies a mild tuning of VEVs to achieve an acceptable neutrino mass pattern. In the indirect models, each column is generated from a unique flavon aligned along a direction corresponding to a column of U_{TB} so there is no tuning required for the neutrino masses, and this is called natural form dominance. In the limit that $m_1 \ll m_2 < m_3$ natural form dominance reduces to constrained sequential dominance.

We have also discussed the theoretical implications of the measurement of a non-zero reactor angle, as hinted at by recent experimental measurements. A measurement of a large reactor angle, consistent with the present 2σ indication for $r = 0.2$, can still be consistent with tri-bimaximal solar and atmospheric mixing, corresponding to $s = a = 0$, according to the tri-bimaximal-reactor mixing hypothesis. This can be achieved in the see-saw mechanism using partially constrained sequential dominance, which may be readily be realized in classes of indirect models based on D-term vacuum alignment.

We have surveyed models based on a Family Symmetry \otimes GUT symmetry structure and noted that direct models tend to be based on $SU(5)$ GUTs while indirect models allow $SO(10)$ GUTs. Another more technical distinction is that direct models are based on F-term vacuum alignment, while indirect models often use D-term vacuum alignment. However the new class of models based on $PSL(2, 7)$ [7] involving sextet flavons (allowing improved top quark Yukawa convergence) may yield a direct model involving D-term vacuum alignment and $SO(10)$.

Finally we have noted that models based on non-Abelian family symmetry can solve the SUSY flavour and CP problems. This increases the motivation for considering models with non-Abelian family symmetry, especially models in which all three entire quark and lepton families (including both left and right-handed components) transform as a triplet under the family symmetry, since such models offer maximum protection against SUSY induced flavour changing and CP violation.

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